WHAT DOES BEING 'IN-TUNE' MEAN? *A brief introduction to temperament*

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What makes music 'music' is the mathematical relationship between the pitch of different notes both in melody and harmony. The simplest possible relationship is the octave, where the ratio of the number of vibrations of a note (string, column of air, reed etc) of two notes one octave apart is two to one. Other simple ones are the fifth, where the ratio is 3/2 or 1.5, and the fourth where the ratio is 4/3 or 1.333. For some reason, we perceive these ratios as pleasing, at least in western music. This concept of ratios is known as 'just' intonation or temperament. On certain instruments, notably the valve-less bugle, the only notes that can be played are related to each other by these simple mathematical ratios.

However, there is a big problem with using ratios. On a piano keyboard, one can go round what is called the 'circle of fifths', whereby starting at one note, say the A above middle C which is usually tuned to vibrate at 440 vibrations per second (cycles per second or Hertz), one can play the fifth above the starting note, in this case E, and then the next fifth (B) and so on, eventually ending up (twelve fifths later) at another A. On a piano, this works just fine, but what happens when the ratio of 3/2 is used? The answer is that one never ends exactly back at an A. The table below shows this in action. To avoid running out of keys, we need to go down an octave every two fifths:

A	440Hz	
E	660	
B	495	
F#	742.50	
C#	556.88	
G#	835.31	
D#	626.48	
A#	939.73	
E# = F	704.79	
C	1057.19	
G	792.89	
D	1189.34	
A	446	*This should be 440, so 6 Hz out*

Now 6Hz doesn't sound a lot but it's actually a quarter of a semitone at this pitch. Just about anyone, no matter how tone-deaf, can hear this amount of out-of-tune-ness. Pythagoras first wrote about this problem, which is why this difference is termed the Pythagorean comma and he came up with a more complex set of ratios to try and solve it. Unfortunately, the values he used for thirds and sixths were so far out of tune that they were unusable, and consequently absent from early polyphonic music.

This problem is particularly noticeable in music where modulation to different keys is required, as a particular note in one key is not necessarily the same pitch in another. Taking A (440) as a reference point and using just temperament, the ratios and frequencies of an A major scale are thus:

Note	**Ratio**	**Ratio** *(as a decimal)*	**Frequency**
A	1		440
B	9/8	1.125	495
C#	5/4	1.25	550
D	4/3	1.333	586.7
E	3/2	1.5	660
F#	5/3	1.667	733.3
G#	15/8	1.875	825
A	2		880

Now say we wanted to be in C# minor, we can go up from the A by a major third (ratio 5/4):

C# = 440 * 5/4 = 550Hz

and then we can go to the leading note B which is a whole tone down from C#:

B = 550 * 8/9 = 515.625Hz

But B is also a whole tone up from A:

B = 440 * 9/8 = 495Hz

Thus the B in A major is not the same as the B in C# minor. One way of correcting this would be to have another B key on the keyboard at the higher pitch, and in fact examples of early instruments exist where the black notes on a keyboard are split for just this situation. Unfortunately it makes playing rather difficult.

It turns out to be impossible to tune the twelve-note chromatic scale so that all intervals are 'perfect', and many different methods of tuning with their own various compromises have thus been used over the years.

The one that has won the day, at least as far as keyboard instruments are concerned from the 19th century onwards, is equal temperament. In this tuning method, all semitones are equal and defined to be 1.05946 (mathematically the 12th root of 2) times the frequency of the semitone below. This can't be expressed as a simple fraction, although in early electronic instruments a set of integer dividers from a very high frequency gave a good approximation. For example, 125015Hz divided by 284 gives A at 440Hz, and divided by 451 gives the C# below. In equal temperament, all keys have exactly the same tuning, every semitone has the same error relative to its neighbour, and thus sound the same irrespective of the key.

Bach's use of the term 'well tempered' was somewhat different in that the tuning errors were less evenly distributed giving distinct characters to different keys. Some musicians talk of these 'key characters', for example bright or tending to flatness, but they can only really apply to music where modern-tuned instruments are not involved.

The table below shows the difference between equal and just temperaments. Errors in tuning are usually specified in cents which is one twelve-hundredth of an octave or a hundredth of a semitone.

Note	Equal Ratio	Hz	Just Ratio	Decimal	Hz	Error (cents)	Compared with just equal is:
A	1	440	1	1	440	0	
A#	1.0595	466	16/15	1.0667	469	12	flat
B	1.1225	494	9/8	1.1250	495	4	flat
C	1.1892	523	6/5	1.2000	528	16	flat
C#	1.2899	554	5/4	1.2500	550	14	sharp
D	1.3348	587	4/3	1.3333	587	2	sharp
D#	1.4142	662	7/5	1.4000	616	17	sharp
E	1.4983	659	3/2	1.5000	660	2	flat
F	1.5874	699	8/5	1.6000	704	14	flat
F#	1.6818	740	5/3	1.6667	733	16	sharp
G	1.7818	784	9/5	1.8000	792	18	flat
G#	1.8876	831	15/8	1.8750	825	12	sharp
A	2	880	2/1	2	880	0	

This shows that the same sequence of notes played on a piano and played on a bugle or sung by a singer will not necessarily be identical in pitch. The differences are small for major 2nds, 4ths and 5ths, but quite large for 3rds, 6ths and 7ths.

There's one further complication with just temperament in that the semitone difference between a major and a minor third (C and C# in the above table) is 25/24 and the difference between semitones at either end of the scale (A and A# and G# and A) is 16/15 which is a huge variation. In cents this is 70 and 112 respectively making the two semitones 42 cents different which is nearly a quarter of a tone. Also there are other pretty good ratios such as 16/9 for a minor 7th which is only 4 cents away from equal, but is of a higher order than 9/5. This can give 'just' singers or bugle players more than one way of getting to the 'same' note.

Harmony is also based on natural values, so a chord of A major would be vibrating in the ratios 4 : 5 : 6 (ignoring the octave):

A 440Hz
C# 440 * 5/4 = 550Hz
E 440 * 3/2 = 660Hz
A 440 * 2 = 880

All nice round numbers, which gives a pure sound, and there are no beat notes as all the (just) harmonics are in tune with each other. The same chord played on a piano will have these frequencies:

A 440Hz
C# 440 * 1.2599 = 554.4Hz Very sharp
E 440 * 1.4983 = 659.3Hz Very slightly flat
A 440 * 2 = 880

This will result in very characteristic beat notes between the harmonics of the strings which are not in tune with each other. The real difference between just and equal temperaments often leads choir directors, especially ones who have spent their formative years as piano or organ players, to try to make a choir sing major thirds and leading notes higher (brighter) to make the choir match the pitch characteristics of a (equal tempered) keyboard.

In summary, it's not surprising that singers who rely on intervals and naturally sing 'just' intervals tend to go 'out of tune', especially in chromatic music as they have no solid frame of reference. Instrumentalists on fretless stringed instruments at least have the open string as their reference, but they will be continually re-tuning as they play, to keep the intervals just and the harmonies natural. In an *a cappella* choir that does stay in tune, it may be due to presence of one or two singers with perfect pitch. It's also possible that pitch memory is involved where some singers can keep their own local frame of reference going for long enough to keep the tuning intact.

It also begs the question, what is meant by perfect or absolute pitch when applied to a singer? A quick straw poll of three cathedral lay clerks who do have perfect pitch, gave the result that they have equal temperament pitch. This means they actually sing the same notes as played on a piano, which suggests that in perfect pitch the pitches are remembered.

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